

Using A Fuzzy Regression Discontinuity Designs (Ford) To Estimate Treatments When There Is Insufficient Information About Cut Point Selection: A Review

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Abstract

The study aimed to identify and comprehensively review studies related to the use of Fuzzy Regression Discontinuity Designs (FRD) to estimate treatments when there is insufficient information about choosing the cutoff point with reference images illustrating this design. Fuzzy logic and fuzzy inference systems have been used to incorporate imprecise or vaguely-defined information data. fuzzy regression discontinuity design is used to identify the causal effect Fuzzy sets have also been frequently used to model non-precise statistical data . In fuzzy regression discontinuity (FRD) designs, the treatment effect is identified through a discontinuity in the conditional probability of treatment assignment. We show that when identification is weak (i.e. when the discontinuity is of a small magnitude) the usual t-test based on the FRD estimator and its standard error suffers from asymptotic size distortions as in a standard instrumental variables setting. Literature Review has focused on how to use a fuzzy discontinuity regression design to estimate the treatment effect. To eliminate the asymptotic size distortions that the standard error suffers from . Has been reached a new set of testable implications, characterized by a set of inequality restrictions on the joint distribution of observed outcomes and treatment status at the cut-off. We propose a nonparametric test for these testable implications. The test controls size uniformly over a large class of distributions of observables, is consistent against all fixed alternatives violating the testable implications, and has nontrivial power against some local alternatives.

Keywords: Fuzzy Regression Discontinuity Designs ; nonparametric test ; Estimating effect of treatment .

Introduction :

To deal with endogeneity problems arising from partial compliance, as is standard in the literature, we follow an instrumental variable estimation strategy by using the exogenous assignment to the treatment as an instrument for effective participation in the remedial courses. Therefore, we use a fuzzy regression discontinuity design in which the treatment status is probabilistically determined as a discontinuous function of the test score.

In fuzzy regression discontinuity (FRD) designs, the treatment effect is identified through a discontinuity in the conditional probability of treatment assignment. We show that when identification is weak (i.e. when the discontinuity is of a small magnitude) the usual t-test based on



the FRD estimator and its standard error suffers from asymptotic size distortions as in a standard instrumental variables setting (Feir, et al., (2016)). This problem can be especially severe in the FRD setting since only observations close to the discontinuity are useful for estimating the treatment effect. To eliminate those size distortions.

Many empirical studies use Fuzzy Regression Discontinuity (FRD) designs to identify treatment effects when the receipt of treatment is potentially correlated to outcomes. Existing FRD methods identify the local average treatment effect (LATE) on the subpopulation of compliers with values of the forcing variable that are equal to the threshold (Hausman, 1978).

Problem of Study:

How to determine treatment status as a discrete function of test score using a fuzzy regression discontinuity design in which the treatment effect is determined by a discontinuity in the conditional probability of being assigned to treatment when specificity is weak.

The aim of study:

The aim is to discuss several key approaches merging fuzziness and stochastic uncertainty in econometrics and statistics and clarify their potential use for future research in this area.

The importance of studying

The importance of the study stems from the fact that the fuzzy regression discontinuity design requires a sharp discontinuity in the independent variable (such as age, income, or any other independent variable) to estimate the effect. This sharp discontinuity is a border point separating two different groups. When designing a study using this pattern, there must be a significant change in the independent variable at the border point of the effect we want to measure. A sharp discontinuity can be used to estimate the effect of a particular policy or intervention on the dependent variable. Fuzzy discontinuity can be used in a variety of fields such as economics, education, and health. The fuzzy regression discontinuity design was searched for research published between 1965 and 2024. The systematic review included forty articles based on the highest pre-specified selection criteria.

Fuzzy regression discontinuity (FRD) designs.

It is a method used to determine the effects of treatment when receiving treatment is associated with outcomes in ambiguous designs. Where the probability of treatment is discontinuous at the cutoff, but not to the point of a final jump from (0-1). To calculate the effect of the treatment, we thus need to review the characteristics that can be observed in the results to obtain the discontinuity of the result. We also review the characteristics that can be observed on the treatment index to obtain (treatment interruption).

Fuzzy regression was not initially introduced as a statistical model. However, the difference between fuzzy regression, fitting curves with fuzzy data, and estimating regression models with fuzzy data has not always been clear, and it has been repeatedly argued that fuzzy regression can replace standard regression models, and even “works better,” in Under different circumstances.

Fuzzy sets have been frequently used to model imprecise statistical data. Fuzzy data arises as a generalization of interval data or, in higher dimensions, of specific value data. In order to formalize the blurring of fundamentally imprecise element boundaries. Meaning it is not rated by one point. Definite value data in statistics and econometrics have been used through random set theory to deal with convex particles, cell shapes, partial determination, and multivariate risk measures.



Vagueness is also used to represent imprecision or ambiguity, for example, in the context of linguistic variables or expert assessments. In practice, most fuzzy subsets are a generalization of intervals, both conceptually and methodologically.

Many experimental studies use fuzzy regression discontinuity (FRD) designs to determine treatment effects when receiving treatment is associated with outcomes. Existing FRD methods determine the local average treatment effect (LATE) on the subpopulation of compliers with effect variable values equal to a threshold.

literature review :

When highlighting the most important developments in fuzzy regression discontinuity (FRD) designs over the past years, there are several important studies in this area . Hundreds of recent applied researches have used that

Fuzzy sets were introduced in Zadeh (1965) as mathematical objects defined via a membership function that generalizes the characteristic function of a set by associating each element of a class with a grade ranging between 0 and 1. The concept of fuzzy set has been long related to fuzzy logic and fuzzy systems, since their original use regards largely to that area (Mamdani and Assilian, 1975, Takagi and Sugeno, 1985 and Zadeh, 1973, Zadeh, 1975). However, the flexibility of considering gradual transitions as opposite to abrupt changes has made the idea flourish fast in a variety of problems where full membership/non-membership is sometimes too restrictive, such as clustering or decision-making (Bezdek, 1981, Bellman and Zadeh, 1970, Ruspini, 1969 and Zimmermann, 1976).

Fuzziness is also used to represent imprecision or vagueness, e.g., in the context of linguistic variables (Zadeh, 1975) or expert ratings (González-Rodríguez et al., 2012). In practice, most of the fuzzy subsets of are a generalization of intervals, both conceptually and methodologically (Ferraro et al., 2010).

Fuzzy linear regression was introduced in 1980 by H. Tanaka, S. Uejima, and K. Asai. The general form of regression equations for fuzzy numbers was developed. They explained that the fuzzy regression problem can be formulated as a mathematical programming problem. The special case of linear regression produces a linear programming problem. Different measures of ambiguity have been introduced to determine the best estimate.

Among the various existing frameworks to handle fuzzy data associated with a standard random experiment, those with more impact relate to the concept of fuzzy random variable introduced by Puri and Ralescu (1986)). This concept has generated a full body of statistical literature (Colubi et al.,2011, Körner, 2000 and Näther, 1997). These statistical tools have been frequently used in areas such as insurance, blind testing or psychology.

Comparisons between regression discontinuity estimators and estimators based on non-confounding or extraneous assumptions were studied.. In the closely related setting of linear instrumental variables models with constant coefficients, such comparisons are often based on Hausman tests (Hausman, 1978). In the local average treatment (LATE) setting with heterogenous treatment effects (Imbens and Angrist (1994), Angrist et al., (1996) , Angrist (2004) discusses the interpretation of the Hausman test and suggests a more attractive test for homogeneity of treatment effects in that context. Extension of the Hausman and Angrist tests to the fuzzy regression discontinuity (FRD)) settings is discussed. Using the FRD developed by Hahn et al. , (2001) and allowing for heterogeneous treatment effects



Fuzzy regression discontinuity (FRD) designs are becoming increasingly important in applied economics. Hundreds of recent applied papers have in many cases used Fuzzy Regression Discontinuity (FRD) designs. The seminal work of Bond et al. (1995) and Staiger and Stock (1997) make weak specificity in the context of instrumental variables (IV) an important consideration in applied work (Stock et al. (2002)) and Andrews and Stock (2007) for literature surveys). However, despite the close parallel between the IV setting and FRD design (Hahn et al. (2001)) there has been no theoretical or practical attempt to deal with poor specificity in FRD design more broadly.

Angrist and Lavy (1999) used the discontinuity of class size with respect to enrollment due to Maimonides' rule to identify the causal effect of class size on student performance. We do not find statistically significant violation of our new testable implication for FRD-validity for any of the four outcome variables (Grade 4 Math and Verb, Grade 5 Math and Verb). In contrast, the existing continuity test suggests statistically significant evidence for discontinuity of the running variable's density at the cut-off (Otsu, et al. (2013)).

Miller, et al. (2013) evaluated the impact of "Colombia's Régimen Subsidiado"—a publicly financed insurance program—on 33 outcomes, where program eligibility is determined by a poverty index. Since our approach makes use of observations of not only the running variable but also of treatment status and the observed outcome, it has the unique feature of being outcome-specific, that is, when multiple outcomes are studied within the same FRD design, researchers can assess credibility of FRD-validity separately for each outcome variable. In this example, the continuity test supports continuity of the running variable density at the cut-off, while we find statistically significant evidence for the violation of our new testable implication for FRD validity for 3 outcome variables (Household Education Spending, Total Spending on Food, and Total Monthly Spending). This result suggests further investigation would be beneficial for identifying and estimating the causal effect on these outcomes .

Fuzzy sets have also been frequently used to model non-precise statistical data (Denœux, 2000, Diamond, 1988, González-Rodríguez et al., 2012 and Viertl, 2011). This is one of the most successful uses of fuzzy sets in the area. Fuzzy data arise as a generalization of interval data, or in higher dimensions, of set-valued data, in order to formalize the blurriness of the boundaries of elements that are essentially imprecise, in the sense of not being constrained to a single point in (Ferraro et al., 2010).

Fuzzy logic and fuzzy inference systems have been used to incorporate imprecise or vaguely-defined information, such as natural language concepts, in decision-making problems or knowledge-based forecasting in economics (Chen et al., 2006 and Cheng et al., 2013).

Other relevant uses of the concept of fuzziness in econometrics and statistics can be found in the context of clustering, where the membership to a cluster is not sharp, or regression-discontinuity designs (Bezdek, 1981, Calonico, et al., 2014, Hahn, et al., 2001 and Yang et al., 2006), where there is some blurriness around the threshold determining who is eligible for certain treatment.

Set-valued or interval data can either be considered the (complex) statistical data of interest per-se, or the imperfect observable outcomes of some underlying precise un-observed statistical data of interest (Manski and Tamer, 2002 and Ramos-Guajardo et al., 2013). The latter is sometimes called epistemic approach, while the former is called ontic approach (Colubi and González-Rodríguez, 2015).



These statistical tools have been frequently used in areas such as insurance, blind testing or psychology, to name but a few (Coppi, et al., 2006b, Ramos-Guajardo, et al., 2019 and Shapiro, 2009). For instance, the space of fuzzy sets has been proposed as a rich alternative to Likert scales in order to capture more information in intrinsically imprecise data, like evaluations, medical diagnoses or quality ratings (González-Rodríguez et al., 2012 and Lubiano et al., (2016) .

Feir, et al., (2016) show that in fuzzy regression discontinuity (FRD) designs, the treatment effect is determined by a discontinuity in the conditional probability of being assigned to the treatment. That is, when specificity is weak (i.e., when the discontinuity is small in size), the usual t-test depends On the FRD estimator and its value.

In fuzzy regression discontinuity (FRD) designs, the treatment effect is identified through a discontinuity in the conditional probability of treatment assignment. We show that when identification is weak (i.e. when the discontinuity is of a small magnitude) the usual t-test based on the FRD estimator and its standard error suffers from asymptotic size distortions as in a standard instrumental variables setting. This problem can be especially severe in the FRD setting since only observations close to the discontinuity are useful for estimating the treatment effect. To eliminate those size distortions, we propose a modified t-statistic that uses a null-restricted version of the standard error of the FRD estimator. Simple and asymptotically valid confidence sets for the treatment effect can be also constructed using this null-restricted standard error. An extension to testing for constancy of the regression discontinuity effect across covariates is also discussed .

Lee (2008) imposes a stronger set of identifying assumptions that implies continuity of the distributions of the running variable and covariates at the cut-off. Following his approach, researchers routinely assess the continuity condition by applying the tests of McCrary (2008), Otsu et al., (2013), Cattaneo et al., (2020), and Canay and Kamat (2018). When the running variable is manipulated, Gerard et al., (2020) provides a partial identification approach in the presence of “one-sided manipulation.” As noted by McCrary (2008), however, in the absence of Lee’s additional identifying assumption, the continuity of the distributions of the running variable and baseline covariates at the cut-off is neither necessary nor sufficient for FRD-validity, and rejection or acceptance of the existing tests is not informative about FRD-validity or violation thereof .

Erendira Leon (2022) published a study on fuzzy regression continuation design titled Timing Compulsory Schooling Impacts on Labor. Market Outcomes in Mexico . Fuzzy Regression Discontinuity Design (RDD) with parametric and non-parametric analyses in UK Statistics Conference 2022.

It dealt with highlighting the effects of compulsory education of 1993 on work. Market results in Mexico: earnings and sectoral options for employment . Raise compulsory school-leaving age from 12 to 15 years. Encourage children to accumulate human capital .The fuzziness addresses imperfect compliance with the policy and use the random assignment of the exposure to the policy.

Marco et al., (May 2023) in their paper (Erasmus Program and Labor Market Outcomes: Evidence from a Fuzzy Regression Discontinuity Design Design) studied the effect of participation in the Erasmus program on a number of labor market outcomes. By implementing a fuzzy regression discontinuity design, participation in an international mobility program was shown to positively affect the probability of employment three years after graduation and reduce the time



taken to find a job, while no significant effect was found on the probability of obtaining a job. The job must be consistent with the qualifications acquired. These results are mainly driven by males and STEM graduates. They also investigated potential mechanisms behind the findings and found that spending a period of time studying abroad improves both spoken English proficiency and graduates' academic performance, and tends to increase the willingness to move to find a job.

Discuss the results:

The aim was to review ways of dealing with fuzziness and imprecision represented by (fuzzy) sets in econometrics and statistics. A review of studies indicated a new test of the validity of the FRD. We first derive a new set of testable effects, characterized by a set of inequality constraints on the joint distribution of observed outcomes and the treatment condition at the maximum. We show that these testable effects are sharp necessary conditions for the validity of the FRD, in the sense that they exploit all the information in the data distribution that is useful for refuting the validity of the FRD.

Local average treatment effects (LATE) are specified in fuzzy regression discontinuity (FRD) designs and comparisons between regression discontinuity estimators and estimators based on non-confounding or exogenous assumptions are studied. In the close setting of linear instrumental variables models with fixed coefficients, these comparisons often rely on Hausman tests in the local average treatment setting (LATE) with heterogeneous treatment effects.

A nonparametric test was used for these testable effects. The test controls for magnitude uniformly over a large class of observable component distributions. We also note that fuzzy regression discontinuity designs are of increasing importance in applied economics, as hundreds of recent applied research have been used in fuzzy regression discontinuity designs.

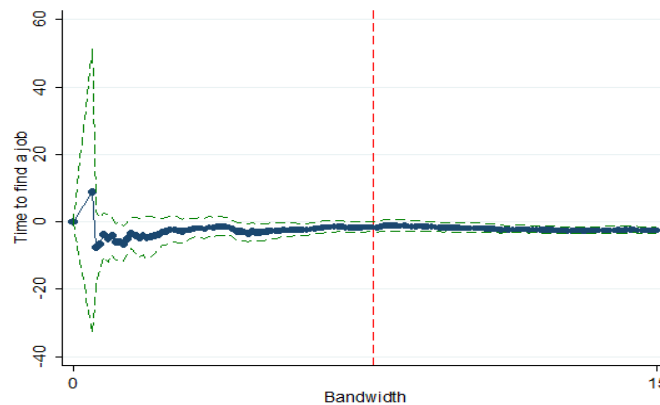
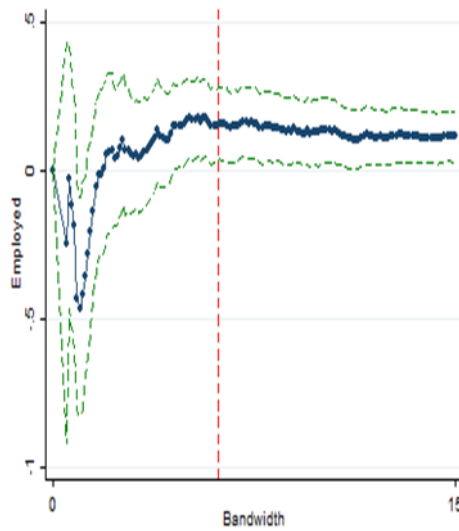
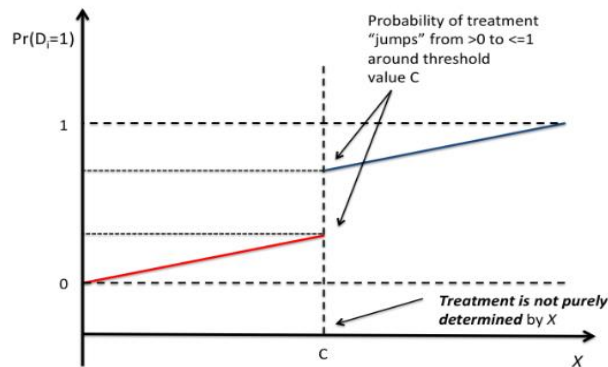
The FRD approach uses observations not only of the current variable. When studying multiple outcomes within the same (FRD) design, researchers can evaluate the reliability of the (FRD) separately for each outcome variable. Thus, the result will be useful for determining and estimating the causal effect of the results. Also, the frequent use of fuzzy sets to model statistical data is one of the most successful uses, as fuzzy sets are created as a design for interval data or in higher dimensions with a specified value in order to formalize the fuzziness of the boundaries of elements that are essentially imprecise, that is, not restrict them to a single point.

Conclusions and recommendations

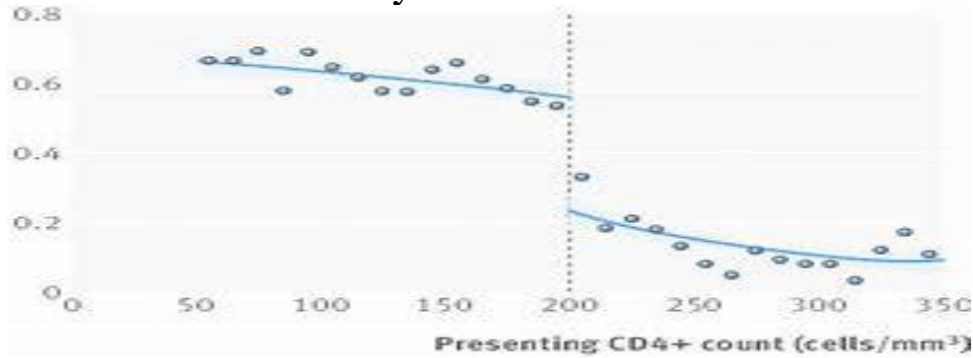
This study demonstrates that Fuzzy Regression Discontinuity Designs (FRD) are an extremely useful tool when the relationship between the independent and dependent variables discontinues, but the assignment to treatment or control groups is not entirely determined by the threshold. Using a regression analysis, we can estimate the causal effect of the independent variable on the dependent variable by introducing some degree of randomness or noise into the assignment. We first derive a new set of testable implications, characterized by a set of inequality restrictions on the joint distribution of observed outcomes and treatment status at the cut-off. We show that these testable implications are sharp necessary conditions for FRD-validity in the sense that they exploit all the information in the distribution of data useful for refuting FRD-validity. We propose a nonparametric test for these testable implications. The test controls size uniformly over a large class of distributions of observables, is consistent against all fixed alternatives violating the

testable implications, and has nontrivial power against some local alternatives. We also suggest conducting another study on FRD and the factors affecting it .

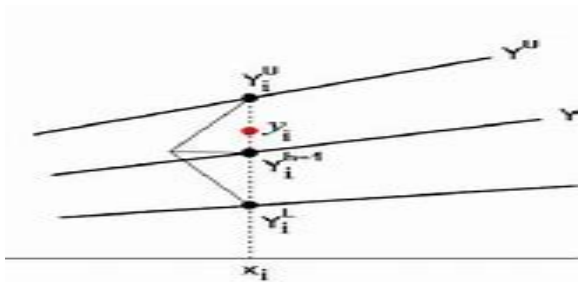
Discontinuity in Treatment: Fuzzy RDD Visually



Fuzzy RDD estimates

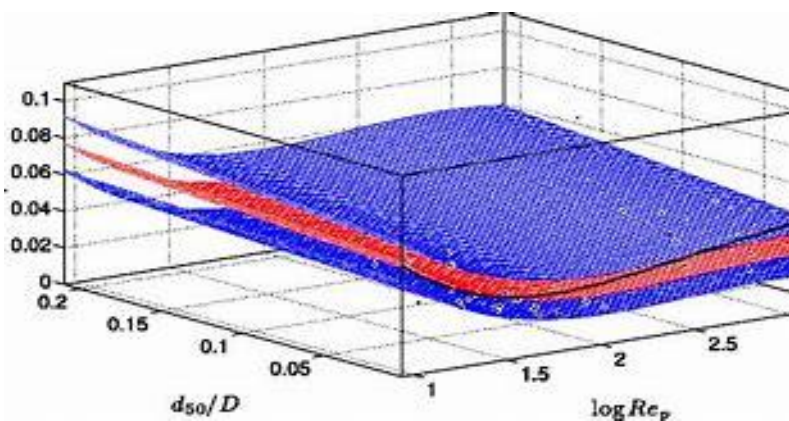
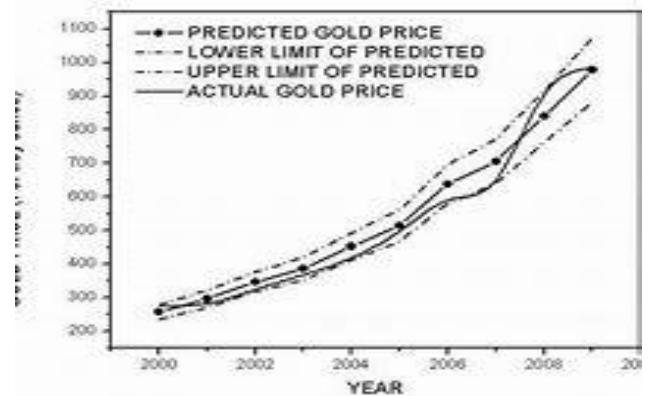


Fuzzy RDD estimates.



Fuzzy regression models

Estimated fuzzy linear regression model with extension



Curves based on fuzzy regression are derived, according to the proposal

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