The Peerian Journal
Open Access | Peer Reviewed
Volume 51, February, 2026
Website: www.peerianjournal.com
ISSN (E): 2788-0303
Email: editor@peerianjournal.com

Development of a mathematical model and selection of optimal parameters for neutralizing wastewater via the electrohydraulic effect

Nurlan Aytbaev
Tashkent state technical university named after Islam Karimov, Tashkent, Uzbekistan.
Corresponding author: aytbaev.nurlan@mail.ru

Abstract. This study explores the application of the electrohydraulic effect as an alternative method for disinfecting wastewater. It details the construction of a laboratory-scale setup and outlines the interchangeable components involved in the treatment process. A mathematical model has been developed to simulate the electrohydraulic treatment, accompanied by an optimization study of key parameters such as discharge voltage, capacitor capacity, and treatment duration. These variables were analyzed through statistical experimental design methods. A second-order regression model was formulated to predict disinfection efficiency, and its validity was confirmed using Student's *t*-test and Fisher's *F*-test. Optimal operating conditions were established, achieving high treatment efficiency while maintaining eco-friendliness and energy conservation. In addition to conventional disinfection methods, this work emphasizes the importance of innovative technologies in modern wastewater treatment research.

Introduction

Modern wastewater treatment technologies include mechanical, chemical, physico - chemical, and biological methods, as well as their combined applications [1]. The complexity of choosing the optimal treatment method is due to the heterogeneity of pollutants in wastewater and the need to achieve high purification efficiency [2].

Protecting the environment and using natural resources wisely is one of the main issues of our time. In the treatment and neutralization of wastewater, the application of energy- and resource-efficient technologies and technical means plays a leading role. Globally, approximately 1.04 billion cubic meters of wastewater are generated daily. Of this, the industrial sector accounts for about 22–25%, the residential sector (domestic waste) for about 50%, and agriculture (especially irrigation runoff) for about 25–30%. To treat and neutralize this water, it is necessary to introduce environmentally friendly, energy-efficient, and high-quality machines into practice[1]. In this regard, the use of high-performance and energy- and resource-efficient technical tools and equipment for wastewater treatment is of great importance.

Environmental pollution and the rational use of natural resources are critical issues today. One of the effective methods for addressing these problems is the energy-efficient treatment of wastewater before it is discharged into natural water bodies [3]. The electrohydraulic effect - a process

---

[1] https://www.unesco.org/reports/wwdr

involving high-voltage pulsed discharges in water - has shown promising results for wastewater neutralization due to its eco-friendliness and high efficiency.

An analysis of scientific studies on wastewater neutralization using the electrohydraulic effect showed that, under soft operating conditions [4], it is advisable to select the voltage and capacitor capacity within the range of U<20 kV, C>1,0 mkF;

Theoretical Research

The combined effect of these factors leads to a sharp decrease and elimination of the microorganism population.

The effectiveness of microorganism reduction, or the degree of neutralization (η), as a result of the electrohydraulic effect (EHE), is usually expressed by the following equation [5]:

$$\eta = \frac{N_0 - N_a}{N_0} \qquad (1)$$

N_0 - the initial number of microorganisms, N_a – the number of surviving microorganisms after treatment with the electrohydraulic effect (EHE).

In the process of wastewater neutralization, the main factors used to evaluate the effect of electrohydraulic treatment are discharge voltage (U), treatment time (τ), and capacitor capacity (C). These factors are influenced by various electrical and technological conditions. To study their combined effect, it is necessary to determine the functional relationship between the parameters of the treated wastewater and the electric pulsed discharge.

To reduce the number of experiments and increase accuracy, as well as to derive mathematical equations describing the process and determine the optimal regime parameters within the research domain, the study was conducted using the mathematical theory of experimental design [6, 7].

To construct the experimental design matrix, the transition from actual (natural) values of the factors to their coded (dimensionless) values was carried out using the following formula:

$$x_i = \frac{X_i - X_{i0}}{\varepsilon} \qquad (2)$$

where: x_i - the coded value of the *i-th* factor; X_i -the actual (natural) value of the *i-th* factor; X_{io} - the central (zero-level) value of the *i-th* factor; ε-the variation interval of the corresponding factor.

For each factor, we first determine the zero level and the range of variation, and then proceed with coding.

We choose the following type of mathematical model:

$$y = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i<j}^{n} b_{ij} x_i x_j + \sum_{n=1}^{n} b_{ij} x_i^2 \qquad (3)$$

We use a second-order experimental design method of the Bn type.

From preliminary experiments, the following were identified as the main parameters affecting the degree of wastewater neutralization (η):

X1 – Discharge voltage, V
X2 – Capacitor capacity, μF
X3 – Treatment duration, s.

During the experiments, three trials were conducted at each point of the B3 design spectrum. The order of the experiments was carried out according to the randomization table [6]. The selected factors, their variation intervals, and levels are presented in Table 1.

Table 1. Factor variation intervals and levels

Designation of Factors		Factor	Interval	Levels		
Coded	Natural			-1	0	+1
X1	U	Discharge voltage, kV	1	8	9	10
X2	C	Capacitor capacity, mkF	0,1	1,2	1,3	1,4
X3	τ	Treatment duration, s	1,5	5	6,5	8

The matrix of the basic functions for this design is presented in Table 2. The average values and variances of the *m* parallel experiments in Table 2 were calculated using the following formulas:

$$\bar{y}_g = \frac{1}{m}\sum_{i=1}^{m} y_{gi} \tag{4}$$

$$S_g^2 = \frac{1}{m-1}\sum_{i=1}^{m}\left(y_{gi} - \bar{y}_g\right)^2 \tag{5}$$

The reproducibility of the experiment is verified using Cochran's criterion. We check the reproducibility of the experiment according to Cochran's criterion for: $q = 0{,}05$

$$G = 0{,}01064 < G_{1-q}\left(v_1 = 2,\ v_2 = 28\right) = 0{,}2354$$

This value does not contradict the hypothesis of the experiment's reproducibility based on the observed results.

The degree of reproducibility of the variance is calculated using the following formula:

$$S^2\{y\} = \frac{1}{N}\sum_{g=1}^{N} S_g^2 = \frac{1}{N_1 + 2n + N_0}\sum_{g=1}^{N} S_g^2 = \frac{94}{14} = 6{,}71429 \tag{6}$$

The average variance is calculated as follows:

$$S^2\{\bar{y}\} = \frac{S^2\{y\}}{m} = \frac{6{,}71429}{3} = 2{,}2381 \tag{7}$$

To determine the regression coefficients, the following sum is calculated:

$$z_j = \sum_{g=1}^{N} f_{gj}\bar{y}_g, \quad j = 0\ldots14 \tag{8}$$

Where: $j = 0$ for $f_{g0} = 1$; $j = 1\ldots4$ for $f_{gj} = (x_i)_g$; $j = 5\ldots10$ for $f_{gj} = (x_i x_j)_g\ (ij = 1\ldots4,\ i = f)$; $j = 11\ldots14$ for $f_{gj} = (x_i^2)_g\ (i = 1\ldots4)$.

The calculations are presented in Table 2.

TABLE 2. Experimental Design Matrix of B_n (B_3) Type and Experimental Results

g	Matrix of Basis Functions F										y_{g1}	y_{g2}	y_{g3}	$\bar{y}_g$	S_g^2	$\hat{y}_g$
	$f_0(\underline{x})$	$f_1(\underline{x})$	$f_2(\underline{x})$	$f_3(\underline{x})$	$f_5(\underline{x})$	$f_6(\underline{x})$	$f_8(\underline{x})$	$f_{11}(\underline{x})$	$f_{12}(\underline{x})$	$f_{13}(\underline{x})$						
	1	x_1	x_2	x_3	x_1x_2	x_1x_3	x_2x_3	x_1^2	x_2^2	x_3^2						
1	-1	-1	-1	+1	+1	+1	+1	+1	+1	-1	73	71	74	72,6	2,3	71,7
2	+1	-1	-1	-1	-1	+1	+1	+1	+1	+1	79	81	82	80,6	2,3	80,46
3	-1	+1	-1	-1	+1	-1	+1	+1	+1	-1	73	76	74	74,3	2,3	73,6
4	+1	+1	-1	+1	-1	-1	+1	+1	+1	+1	86	83	87	85,3	4,3	84,93
5	-1	-1	+1	+1	-1	-1	+1	+1	+1	-1	80	81	83	81,3	2,3	81,73
6	+1	-1	+1	-1	+1	-1	+1	+1	+1	+1	93	84	92	89,6	24,3	90,3
7	-1	+1	+1	-1	-1	+1	+1	+1	+1	-1	88	86	84	86	4	86,2
8	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	98	97	94	96,3	4,3	97,3
9	-1	0	0	0	0	0	+1	0	0	-1	79	81	74	78	13	79,03
10	+1	0	0	0	0	0	+1	0	0	+1	91	90	89	90	1	88,96
11	0	-1	0	0	0	0	0	+1	0	0	79	83	85	82,3	9,3	82,43
12	0	+1	0	0	0	0	0	+1	0	0	86	85	90	87	7	86,9
13	0	0	-1	0	0	0	0	0	1	0	77	79	72	76	13	78,23
14	0	0	+1	0	0	0	0	0	1	0	90	94	91	91,6	4,3	89,43

Using the calculated sum values for $Z_j(j = 0 \ldots 14)$, the regression coefficients are determined using the following formulas:

$$b_0 = \frac{a}{N}\sum_{i=1}^{N}\bar{y}_g - \frac{b}{N}\sum_{i=1}^{n}\cdot\sum_{g=1}^{N}(x_i^2)_g\bar{y}_g \tag{9}$$

$$b_i = \frac{1}{\lambda_2 \cdot N}\cdot\sum_{g=1}^{N}(x_i \quad)_g\,\bar{y}_g \tag{10}$$

$$b_{ij} = \frac{1}{\lambda_3 \cdot N}\cdot\sum_{g=1}^{N}(x_i x_j \quad)\bar{y}_g \tag{11}$$

$$b_{ij} = \frac{C}{N}\cdot\sum_{i=1}^{N}(x_i^2 \quad)\bar{y}_g - \frac{d}{N}\cdot\sum_{i=1}^{n}\cdot\sum_{g=1}^{N}\ (x_i^2 \quad)\bar{y}_g - \frac{b}{N}\cdot\sum_{g=1}^{N}\bar{y}_g \tag{12}$$

Here: a,b,c,d $(\lambda_2 \cdot N)^{-1}, (\lambda_3 \cdot \mathrm{N}\ )^{-1}$re constants, auxiliary constructions used for calculating model coefficients [8].

When n=3 and the number of coefficients b_{ij} is equal to three, the values of the constants are as follows:
a = 5,6875; b= 2,1875; c= 7; d= 1,3125; $(\lambda_2 \cdot N)^{-1} = 0{,}1$; $(\lambda_3 N)^{-1} = 0{,}125$.

TABLE 3. Results of calculating the t_j-criterion and the b_j coefficients.

basis function	j	z_j	b_j	$S^2(b_j)$	$S(b_j)$	t_j
1	0	1171,33	84,60	0,90923	0,95353	88,73
x_1	1	49,67	4,97	0,22381	0,47309	10,50
x_2	2	22,33	2,23	0,22381	0,47309	4,72
x_3	3	56,00	5,60	0,22381	0,47309	11,84
x_1x_2	4	5,00	0,62	0,27976	0,52893	1,18
x_1x_3	5	-0,33	-0,04	0,27976	0,52893	-0,08
x_2x_3	6	5,00	0,63	0,27976	0,52893	1,18
x_1^2	7	834,33	-0,60	0,90923	0,95353	-0,63
x_2^2	8	835,67	0,06	0,90923	0,95353	0,07
x_3^2	9	834,00	-0,77	0,90923	0,95353	-0,81

The variances of the coefficients are calculated using the following expressions:

$$S^2(b_0) = \frac{a}{N} S^2\{\bar{y}\}; \tag{13}$$

$$S^2\{b_i\} = (\lambda_2 \cdot N)^{-1} S^2\{\bar{y}\}; \tag{14}$$

$$S^2\{b_{ij}\} = (\lambda_3 \cdot N)^{-1} S^2\{\bar{y}\}; \tag{15}$$

$$S^2\{b_{ii}\} = \frac{C - OC}{N} S^2\{\bar{y}\}; \tag{16}$$

The values of the t_j-criterion are calculated using the following expression:

$$t_i = \frac{|b_j|}{S\{b_j\}} \tag{17}$$

where:
$S\{b_j\} = \sqrt{S^2\{b_j\}}$ - sample standard deviation (root mean square deviation).

The significance of the regression coefficients is verified by comparing the null hypothesis with the alternative value of Student's *t*-criterion using the following inequality:

$$t_j > t_{1-\frac{g}{2}}(v = N(m-1)) \tag{18}$$

where:

$t_{1-\frac{g}{2}}(\nu)$ - V = N (m-1) - Student's *t*-distribution quantile with $(1-\frac{g}{2})$ % degrees of freedom;

the null hypothesis $H_0 \cdot \beta = 0$ is rejected, and the corresponding estimate of b_i is considered statistically significant.

In this case, for q=0.05, the quantile of the Student's *t*-distribution $t_{1-\frac{g}{2}}(8) = 2{,}011$

Thus, the mathematical model is obtained in the following form:

$\eta = 84{,}6 + 4{,}97X_1 + 2{,}23X_2 + 5{,}6X_3 + 0{,}62X_1X_2 - 0.04X_1X_3 + 0{,}63X_2X_3 - 0{,}6X_1^2 + 0{,}06X_2^2 - 0{,}77X_3^2$ The next stage in processing the experimental results is to test the hypothesis regarding the adequacy of the mathematical model and the response function. After regression analysis, this is done by comparing the sample variance and the adequacy variance. The hypothesis about the adequacy of the model - specifically, whether the two variances are equal - is tested using Fisher's criterion.

$$F = \frac{S^2_{OTK}}{S^2\{\bar{y}\}} \qquad (19)$$

The selected variance is calculated using the following formula:

$$S^2_{OTK} = \frac{\sum_{g=1}^{N}(\bar{y}_g - \hat{y}_g)^2}{N-d} \qquad (20)$$

The calculation is performed based on the value from the table:

$$S^2_{OTK} = \frac{15{,}2889}{4} = 1{,}7078$$

$S^2_{OTK} > S^2_{\{\bar{y}\}}$ Taking n into account, the calculation is performed as follows:

$$F = \frac{S^2\{\bar{y}\}}{S^2_{OTK}} = \frac{2{,}2381}{1{,}7078} = 1{,}31051$$

V_1=N-d=14-10=4; V_2=N(n-1) =14(3-1) =28

At q=0.05, the tabulated value of the Fisher criterion according to table [7] is as follows:

F= 1,31051 <F_{1-q}(4,28) = 2,71

Thus, the hypothesis regarding the adequacy of the mathematical model and the response function does not contradict the observed results.

By removing the statistically insignificant coefficients and based on the obtained calculation results, the mathematical model in the coded form is as follows:

$$\hat{y}(x,b) = 0{,}474 + 0{,}008x_1 + 0{,}083x_2 - 0{,}008x_3 + 0{,}025x_1x_2 - 0{,}001x_1x_3 + \\ + 0{,}018x_2x_3 + 0{,}053x_1^2 - 0{,}055x_2^2 - 0{,}065x_3^2 \qquad (21)$$

The transition from coded values to natural values of the variables is carried out using the following expression:

$$x_i = \frac{X_i - X_{i0}}{\varepsilon} \qquad (22)$$

According to expression (22), the values of the variables in the equation of the neutralization process using the electrohydraulic effect are as follows:

$$x_1 = \frac{U-9}{1}; \qquad x_2 = \frac{C-1{,}3}{0{,}1} \qquad x_3 = \frac{\tau - 6{,}5}{1{,}5};$$

After converting the coded values to natural values and applying the corresponding transformations, the mathematical model representing the degree of neutralization obtained from the treatment of wastewater with electric pulsed discharge takes the following form:

$$\eta = 40.179247 + 7.8832U + 2.929212\tau - 76.379C + 6.2UC - 0.0264U\tau +$$

$$+ 4.158\tau C - 0{,}6U^2 - 0{,}335412\tau^2 + 6C^2$$

To find the optimal value of the mathematical model, calculations are performed using the PascalABC programming software.

The block diagram of the algorithm for calculating the neutralization efficiency is shown in Figure 1.

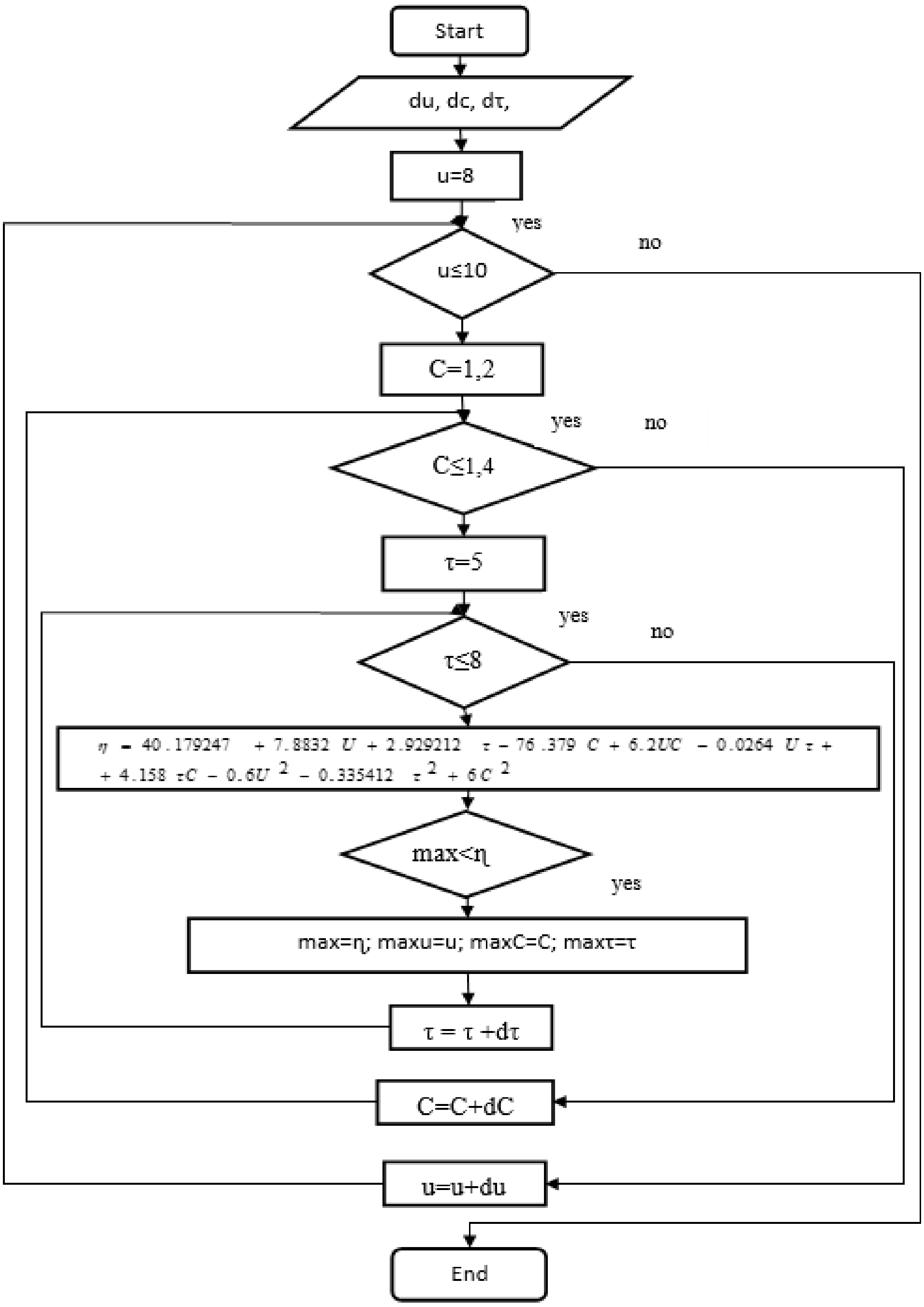

FIGURE 1. – Block diagram of the algorithm for calculating the neutralization efficiency during wastewater treatment using the electrohydraulic effect.

Conclusions

It has been established that the decrease in the concentration of microorganisms during electrohydraulic treatment follows an exponential dependence on energy input, which allows for more accurate prediction of disinfection efficiency

As a result of the research, the following optimal parameters for the wastewater treatment process using the electrohydraulic effect were determined: discharge voltage of 10 kV, treatment time of 8 seconds, and capacitor capacity of 1.3 μF. Under these parameters, the volume of neutralized water (equipment productivity) reached 11 m^3/day.

References

1. Aytbaev, N.A., Berdyshev, A.S., Turdibaev, A.A., Sobirzhonov, N.U. (2023). Electrohydraulic disinfection of liquids and evaluation of efficiency under different regimes. In Innovation in the Modern Education System, Part 30, Collections of Scientific Works. Washington, USA, 25th May 2023.
2. Turdiboyev, A., Aytbaev, N., Mamutov, M., Toshev, T., Kurbonov, N. Study on application of electrohydraulic effect for disinfection and increase of water nutrient content for plants IOP Conference Series: Earth and Environmental Science, 2023, 1142(1), 012027.
3. N. A. Aytbayev, A. S. Berdishev, A. A. Turdibayev. Application of the Electrohydraulic Effect Method for Wastewater Disinfection // Economics and Society, No. 12(127), 2024.
4. Shengqi Zhang, Chengsong Ye, Jianguo Li, Xin Yu, Mingbao Feng Treatment-driven removal efficiency, product formation, and toxicity evolution of antineoplastic agents: Current status and implications for water safety assessment // Water Research. Volume 206, 1 November 2021, 117729. https://doi.org/10.1016/j.watres.2021.117729.
5. Augambaev, M., Ivanov, A., & Terekhov, Yu. (2007). Fundamentals of Scientific Research Experiment Planning: A Textbook. Tashkent: Ukituvchi. 336 pages.
6. Hoshimov, F.A., Bakhadirov, I.I., Kurbanbayeva, M.S., Aytbayev, N.A. Development of specific standards of energy consumption by types of produced products of the spinning product E3S Web of Conferences, 2020, 216, 01169.
7. Borovikov, V. (2001). STATISTICA: The Art of Data Analysis on a Computer. For Professionals. St. Petersburg: Piter. 656 pages.