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Wonderful straight lines and planes of trihedrons

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Abstract. In this article, some information about three-sided angles that are not often found. This article explores the wonderful straight lines and planes of Trihedrons in order for the reader to visualize spatial bodies and form a conic about them.
Key words: Median plane of a trihedron, axis of a circular cone, internal and external cone axes, Angle, inequality, proof, cosines theorem, bisector, angle.sines theorem, center of gravity, plane angle, two-sided corner

Necessary and sufficient conditions for the existence of a trihedron

For the trihedron to exist $(0,\pi)$ was between α, β, γ the following inequalities are necessary and sufficient for flat angles where the magnitudes are:

$$|\beta-\gamma|<\alpha<\beta+\gamma \quad \text{va} \quad \alpha+\beta+\gamma<2\pi \qquad (1)$$

Proof. It is important. This inequality is proved in expressions (2) and (3).

Yet we $a)\ \beta+\gamma\le\pi$ and $b)\ \beta+\gamma>\pi$ Let's consider two cases: $a)$ Kosinus $(0,\pi)$ decreases monotonically between , and sinus takes a positive value $\cos(\beta+\gamma)<\cos\alpha<\cos|\beta-\gamma|=\cos(\beta-\gamma)$ becomes, This inequality is as strong as the following double inequality:

$\cos\beta\cos\gamma-\sin\beta\sin\gamma<\cos\alpha<\cos\beta\cos\gamma+\sin\beta\sin\gamma$, from here $-1<\dfrac{\cos\alpha-\cos\beta\cos\gamma}{\sin\beta\sin\gamma}<1$.

The above inequality

$$\cos A=\frac{\cos\alpha-\cos\beta\cos\gamma}{\sin\beta\sin\gamma} \qquad (2)$$

so for equality $(0,\pi)$ oralig'ida o'zgaruvchi variable between A means that there is a two-sided angle. Two-sided angle A and stuck to it β and γ we make a trihedron by flat angles. According to the first theorem of cosines α_1 the third plane angle satisfies the following condition:

$$\cos\alpha_1=\cos\beta\cos\gamma+\sin\beta\sin\gamma\cos A \qquad (3)$$

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$$\cos\alpha = \cos\beta\cos\gamma + \sin\beta\sin\gamma\cos A \qquad (4)$$

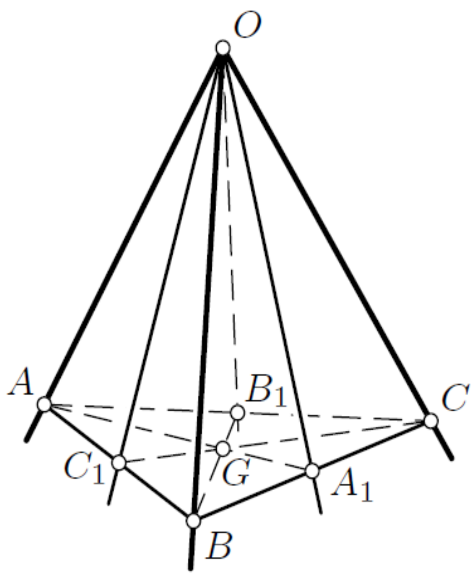
(17) and (18) from Eqs $\cos\alpha_1 = \cos\alpha$ and $\alpha_1 = \alpha$ it follows that Hence the given trihedron α, β, γ has flat corners. has flat corners.

b) $\beta + \gamma < \pi$. (15) dagi $|\beta - \gamma| < \alpha$ and $\alpha < 2\pi - (\beta + \gamma)$ inequalities and this inequality satisfies the following sphere:

$\cos(\beta - \gamma) > \cos\alpha > \cos(2\pi - (\beta + \gamma)) = \cos(\beta + \gamma)$.

This is completed similarly to the proof of the case proved above.

The median plane of the trihedron. The plane passing through the edge forming the trihedron and the bisector of the opposite plane angle is called the median plane of this trihedron.



Theorem 1. Three median planes of a trihedron have a common straight line. (This straight line is called the median of the trihedron).

Proof. The edges of the trihedron are equal to each other starting from the end OA, OB, OC we put the sections and BOC, COA, AOB suitable for flat corners OA_1, OB_1, OC_1 we cross the bisectors. Then it was formed AA_1, BB_1, CC_1

cuts ABC will be the medians of the triangle. If G if the point is the point where they intersect, OG of the light triad OAA_1, OBB_1, OCC_1 median planes consist of a common straight line.

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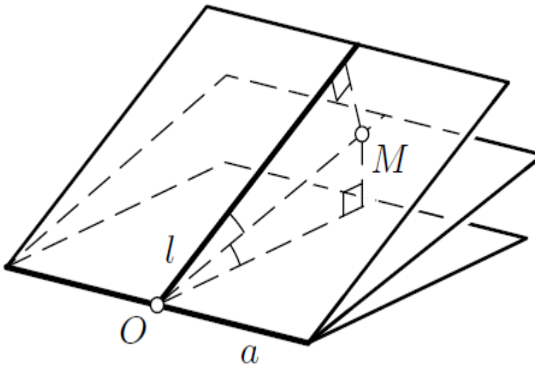
Axis of a circular cone inscribed in a tetrahedron. Let a sided two-sided angle be given. We make one of its linear corners and to it we pass the bisector. a dividing this two-sided angle from the edge into two equal two-sided angles, l The half-plane containing the ray is called the bisector half-plane of the given angle. The plane containing this half-plane is called the plane of the bisector of the two-sided angle.

M a point lies in the plane of the bisector of a two-sided angle when and only if it is equidistant from the planes forming the sides of this angle. In space, in the plane of the bisector of a two-sided angle, there is a set of points equally distant from each side of this angle.



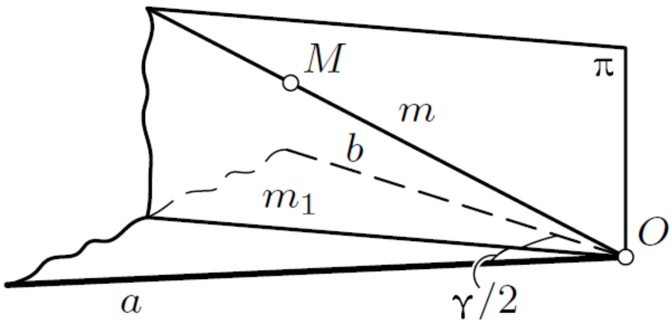
Theorem 2. The bisector planes of the two-sided angles of a trihedron have a common straight line that deviates equal to its sides.

Proof. Two bisector planes of a trihedron intersect p since each point of a straight line is equidistant from its three sides, this straight line lies in the plane of the third bisector of the trihedral. From the equality of congruent right triangles, it can be seen that this straight line forms equal angles with the sides of the trihedron.

In the plane of the sides of the trihedron, its bisector planes intersect p we make orthogonal projections of a straight line. These straight lines form a circular cone surface (circular cone), p and the straight line serves as its axis. This cone is called the internal drawing of this triad. So, the general straight line intersecting the three bisector planes consists of the axis of the circular cone drawn inside this trihedral. A circular cone drawn inside a trihedron is projected onto the plane of its sides by the orthogonal projection of the cone axis.

The axis of the cone drawn outside the trihedron. Three planes perpendicular to the sides passing through the bisectors of the flat angles of the trihedron have a common straight line.

Proof. $Oabc$ Let the trihedral be given. One of the given planes (π plane) and in it O optional from the dot m let's look at the light. This is a flat corner for clarity γ and its bisector m_1 with (m_1 $- m$ of light π relative to the plane Oab orthogonal projection on the plane).



m light $Oabc$ of the triad a va b formula (8) to make sure that the edges form the same angle $Oamm_1$ va $Obmm_1$ we apply to trihedrals. Triedron from two considered planes O common from the point l we will have light. According to the proven property, it deviates equal to the edges of the given trihedron, and therefore lies in the third of these planes.

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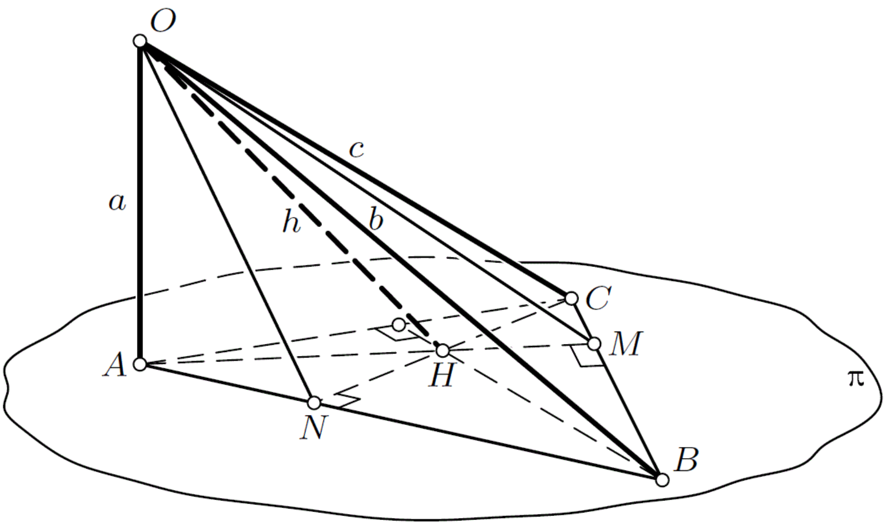
The general straight line intersecting the three planes shown in the theorem forms a circular cone (conic surface) that serves as its axis, and the edges of the trihedron lie on its generator. This cone is called the external cone of this triad.

Height planes and ortho axis of the trihedron. Three planes passing through each edge of the trihedron and perpendicular to the opposite sides are called elevation planes of the trihedron. If none of the edges of the trihedral is perpendicular to the opposite side, these planes are defined as one-valued.

Theorem 3. *The three elevation planes of the trihedron have a common straight line. This straight line is called the ortho axis of the trihedral.*

Proof. $Oabc$ an edge that is not perpendicular to the other edges of the trihedral a we mark with a letter. a perpendicular to the edge and it A intersecting at a point π we will make a plane. This plane is respectively b and c edges B and C intersects at points. BC in a straight line, OAM plain BC perpendicular to the straight line M we get the point. OBC plain OAM perpendicular to the plane BC These planes are perpendicular because they contain a straight line. OAB perpendicular to the plane OCN we make a plane. Then OAM and OCN the planes are the height planes of the given trihedral.



Because NC straight line and OAB because the plane intersects perpendicularly π and OCN plains OAB is perpendicular to the plane. AM and CN and straight lines ABC are the altitudes of the triangle. If $(AM)\cap(CN)=H$ if BH - it follows that this is the third height of the triangle. $OA \perp BH$ and $AC \perp BH$ for being OAC plain BH perpendicular to . that is why OAC and OBH the planes are perpendicular. So, OBH - the third elevation plane of the trihedral, and the straight line is its ortho axis.

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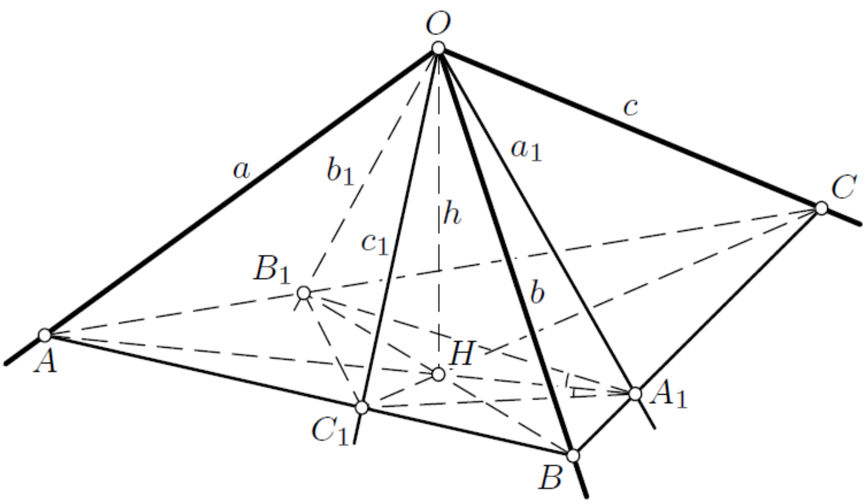
Corollary 1. If π If any of the trihedral edges intersecting a plane is perpendicular to this plane, then it is formed by intersecting points ABC orthocenter of the triangle H lies on the ortho axis of the considered triad.

According to the above result $Oabc$ of the triad an arbitrary edge perpendicular to the plane serves as its height. If ortho axis of the trihedral h If we define with , then π perpendicular to the plane a the edge $Ohbc$ of the triad will be the ortho axis. Therefore, the obtained result can be expressed differently as follows.

Result 2. If the plane is perpendicular to the ortho axis of the trihedron, then the orthocenter of the triangle formed by the intersection points of the edges of the trihedron with the plane lies on the ortho axis of the given trihedron.

Theorem 4. If the ortho axis of the trihedral lies inside it, then the height planes of the trihedral have corresponding sides a_1, b_1, c_1 for a trihedron cut along the rays, these height planes consist of the bisector planes of its two sides.

Proof. $Oabc$ of the triad h perpendicular to the orthosis let the plane be given. According to the 2nd result ABC of the triangle $H = h \cap (ABC)$ has an orthocenter. It is known from planometry, AH , BH and CH rays ABC constructed from the bases of the altitudes of the triangle $A_1B_1C_1$ bisects the angles of the triangle. Thus, the adjacent two-



sided angles are common A_1H according to equation (8) for a straight line B_1A_1H and C_1A_1H of equal angles OA_1B_1 and OA_1C_1 it follows that the angles are equal. $A_1(OB_1H)$ va $A_1(OC_1H)$ corresponding plane angles in trihedrals are equal. Since these angles are acute, according to the theorem of sines (10), the equality of corresponding two-sided angles follows. So this is the general of trihedrals OA_1 two sided angles with an edge are

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equal. Like this $Oabc$ other elevation planes of the trihedral $Oa_1b_1c_1$ bisects the two-sided angles of the trihedral.

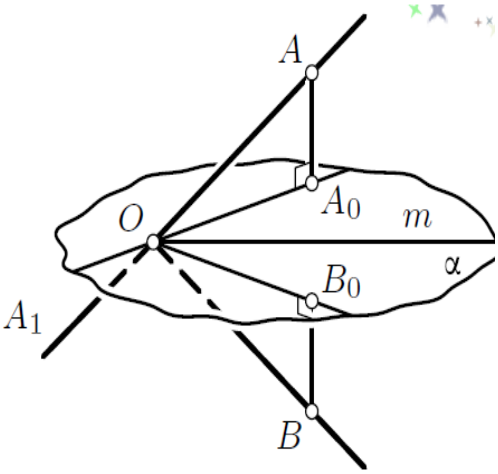
Planes perpendicular to the axis of the cone drawn inside and outside the trihedron

A plane perpendicular to the axis of the cone drawn outside the trihedron. Let's consider a circular cone whose generators are trihedral edges. Its axis forms equal angles with its edges. Let's consider the plane that contains the tip of the trihedron and is perpendicular to the axis of the cone. The edges of the trihedron have the same direction to the plane.

Theorem 5. A cone drawn outside the trihedron is perpendicular to the axis and passes through its tip δ a plane includes the angle bisectors adjacent to its flat corners.

To prove the above theorem, we first prove the following lemma.

Lemma. It is necessary and sufficient that the plane passing through the tip of an angle has the same deviation to its sides, that it contains the bisector of this angle or the bisector of the angle adjacent to it.



Proof. Y e t a r l i l i g. AOB angle bisector m passing through α let the plane be given. in axial symmetry α the plane is self-reflective, OA nur while OB to Because there is movement in symmetry with respect to the axis OA of light α the angle formed by the plane is equal to the angle formed by the ray with this plane. If the plane AOB next to the corner A_1OB passes through the angle bisector, α plain OA_1 and OB it will be the same for the rays. And from that α of the plane OA and OB the rays have the same deviation.

It is important. OA and OB equally deviated from the rays α let the plane be given and let these rays lie in different half-spaces from the plane. OA_0 and OB_0 rays OA and OB of rays α be orthogonal projections on the plane. By condition AOA_0 and BOB_0 angles are equal. If m - A_0OB_0 if it is an angle bisector, m axial symmetry OA_0 the light OB_0 to the light, OAA_0 plain OBB_0 to, OA the light OB reflects to As a result α plain AOB it follows that it passes through the bisector of the angle. If OA and OB rays α if it lies in one of the half-spaces separated by the plane, OB which fills the beam to a straight line OB_1 light α lie in different half-spaces relative to the plane and α the plane will be deviated equal to these rays.

According to the evidence α tekislik AOB passes through the bisector of the angle adjacent to the angle. Let's move on to the proof of the above theorem. the plane should have a deviation equal to the edge of the trihedron, and it should not have a common interior point with the trihedron. By the lemma δ the plane contains the bisectors of the angles adjacent to the plane

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angles of the trihedron. δ In addition to the plane trihedron, there are three other planes that deviate equally from each edge of the given trihedron. These planes are perpendicular to the axes of the cone drawn outward to the triads adjacent to the given triad. Each of them contains two plane angle bisectors of this triad and a plane angle bisector adjacent to the third plane angle.

A plane perpendicular to the axis of the cone inscribed in the trihedron. The axis of the cone drawn inside the trihedron forms equal angles with its sides, and since it forms equal angles with the edges of the trihedron, which is polar to the trihedron, it serves as the axis of the cone drawn outside it. On the contrary, the axis of the cone drawn externally to the trihedral forms equal angles with the sides of the trihedral which are polar to it, and serves as the axis of the cone drawn internally for it.

The plane perpendicular to the axis of the cone inscribed in the trihedron deviates equal to the plane of its sides.

Theorem 6. The axis of the cone drawn inside the trihedron l perpendicular to and passing through its end ω two-sided angles adjacent to the two-sided angles of a plane trihedral lie in the bisector planes and pass through its ends perpendicular to the corresponding edges u_1, u_2, u_3 keeps straight lines.

Proof. Let the trihedral be given. Passing through it a is perpendicular to the edge α we will make a plane. $u_1 = \alpha \cap \omega$ a straight line satisfies the given sphere. Indeed u_1 a straight line a the edge lies in the plane of the bisector of the two-sided angle adjacent to the two-sided corner. Because the bisectors of adjacent two-sided angles are perpendicular to this plane (l, a) is perpendicular to the bisector plane. $\alpha \perp a$ because $u_1 \perp a$ will be. Similarly, for the other two two-sided angles, and ω lying on the plain u_2 and u_3 straight lines satisfy the condition stated in the theorem.

ω in addition to the plane, there are three more planes that are equal to the plane of the sides of the given trihedral and pass through its ends. They are perpendicular to the axes of the cone drawn inside the triads adjacent to this triad.

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